Kinetic Theory Approach to Twin Plane Jets Turbulent Mixing Analysis

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Solutions of twin plane jets based upon a kinetic theory of turbulence are presented. They are obtained by the ensemble averages over the solution of the constructed probability density function in velocity space without using eddy viscosities. The probability density function of the fluid element's motion in the turbulence field is constructed by the integration of a Green's function over the source distributions according to the given boundary conditions. The calculated distributions of the various moments of momentum field are found to be in good agreement with the experimental data. The cross correlation of the fluctuations is described via the revealed joint probability density function of the different components of fluctuation. The behavior of the higher-order turbulent transporting terms is also explained via the distribution of the probability density function of the fluctuation component in the transported direction.

Nomenclature

D	= width of the jets
E	= turbulent energy ($= u'^2 + v'^2 + w'^2$)
F	= probability density function (pdf)
f	= pdf of the fluid element
f_0	= source condition pdf of the fluid element
$f(u'), f(v'), f(w')$	= pdf of the fluctuation velocity in the x, y, and z directions, respectively
$f(u', v')$	= joint pdf of the fluctuation velocity in the x and y directions
G	= Green's function
MP	= merge point
R	= velocity ratio ($\equiv \bar{U}_\infty / \bar{U}_j$)
S	= space of the jets
S_0	= source condition at $x = o$
$\bar{S}$	= space/width ratio ($\equiv S/D$)
t	= time
$\bar{U}_j$	= velocity of the jets at the nozzle exit
$\bar{U}_M$	= mixed center velocity
$\bar{U}_m$	= maximum velocity of the jets
$\bar{U}_0$	= relative velocity ($\equiv \bar{U}_m - \bar{U}_\infty$)
$\bar{U}_\infty$	= surrounding velocity
u, v, w	= instantaneous velocity in the x, y, and z directions, respectively
u', v', w'	= fluctuation velocity in the x, y, and z directions, respectively
$\bar{u}, \bar{v}, \bar{w}$	= average velocity in the x, y, and z directions, respectively
X	= coordinate along the jets' axis
Y	= coordinate of the Y axis
$Y_{1/2}$	= coordinate of the Y axis at $\frac{1}{2}\bar{U}_m$
β_1	= characteristic relaxation rate of the energy-containing eddies
β^ν	= characteristic relaxation rate of the microscale
ω	= chemical reaction term
Λ_1	= characteristic scale of the energy-containing eddies
η	= dimensionless coordinate [$\equiv Y/(x+2D)$]

Subscripts and Superscripts

$\langle\ \rangle$ or —	= ensemble average
∞	= freestream condition
0	= source condition
j, k	= tensor

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Introduction

ANALYSIS of multiple jets is very useful because of its wide application to engineering problems, such as the thrust-augmenting ejectors for VTOL/STOL aircraft, water heaters, and gas stoves for homes. Studies of twin plane jets will provide some basic understanding of the mixing structure of multiple jets.

Many approaches to turbulence modeling have been advanced in recent years. The present work is an alternate approach to turbulence modeling that seeks to describe the statistical characteristics of velocity field. It is a kinetic theory approach that uses a probability density function (pdf) to characterize the stochastic characteristics of fluid elements in velocity space without using the turbulent transport coefficients or eddy viscosities. Past solutions of this statistical model, developed by Chung,[1-4] have generally considered simple one-dimensional geometries and employed the approximate bimodal moment method of Lin and Lees.[5] The Chung kinetic equation was solved by Hong[6] using Green's function to directly integrate the equation for the pdf. In this solution,[6] approximation of the moment method was eliminated. This Green's function method by Hong,[6] in practice, is tedious and difficult to handle in computer calculation.

This method was successfully modified for the free shear layer mixing and combustion problems.[7-9] In the modified Green's function method, the authors actually solved an instantaneous mixing problem to simulate the steady-state phenomenon. The solution will be described further, later in this paper. The previous studies[7-9] demonstrated the inherent advantages of Green's function method for solving Chung's kinetic turbulence equations. Investigations of twin plane jets using Green's function method for construction of the pdf are warranted. This investigation will be further extended to analyze the multiple-jet mixing problem. In the present analysis, the two cases, with and without common end walls, respectively, are considered for calculation (Figs. 1 and 2).

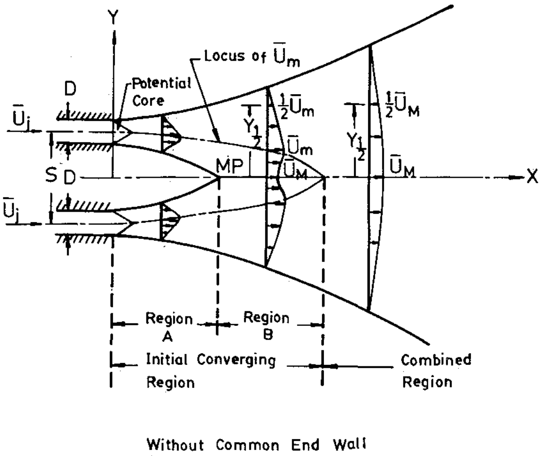

Fig. 1 The flowfield of twin plane jets (without common end wall).

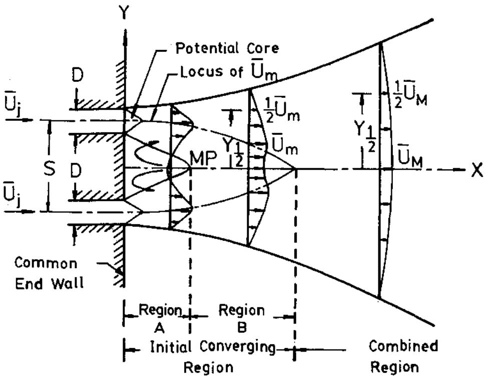

Fig. 2 The flowfield of twin plane jets (with common end wall).

Early experimental work on this problem was carried out by Miller and Comings[10] who studied the plane twin jets with common end wall. Other researchers[11-13] investigated similar problems. However, their investigations were limited to measurements of the lower-order mean quantities such as the mean velocities and mean pressure distributions. Investigation of the higher-order correlations, such as the Reynolds stress and other turbulent transport terms, are scarcely found in the literature. Once pdf is obtained in the present analysis, the higher-order turbulent correlations can be readily constructed.

Theoretical Model and Direct Numerical Solution

Governing Kinetic Equation and the Green's Function

The kinetic theory approach[1-4] utilizes the pdf to describe the turbulent field. The definition of pdf for a fluctuating, turbulent velocity field is analogous to that for molecular velocities. Description of the flowfield of engineering interest is based on the probability density function $f(t,x,u)$ in the present analysis. From the definition of pdf, $f\mathrm{d}u$ represents the probability of finding a fluid element at time t in location x with its instantaneous velocities in the range between u and $u+\mathrm{d}u$.

If the fluid element has a concentration of a scalar quantity of $c(t,x,u)$, then the pdf of this scalar $c(t,x,u)$, $F(t,x,u)$, is $F(t,x,u)=c(t,x,u)\cdot f(t,x,u)$.[1] Derivation of the governing equation for $F(t,x,u)$ is analogous to that of the Boltzmann equation in the kinetic theory of gases. Derivation of the governing equation for F as applied to the twin-jet mixing can be found elsewhere.[1,6,14,15] Assuming that there are no pressure gradients or viscous sublayers, the governing equation for F is

$$\frac{\partial F}{\partial t}+u_j\frac{\partial F}{\partial x_j}=\beta\frac{\partial}{\partial u_j}(u_j-\langle u_j\rangle)F+\frac{\beta_1}{3}E\frac{\partial^2 F}{\partial u_j\partial u_j}+\omega f \tag{1}$$

where

$$\beta=\beta_1+\beta^\nu,\qquad F=fc,\qquad F_i=fc_i,\qquad \Sigma F_i=f\Sigma c_i=f$$

In the present analysis, we assume that there is no chemical reaction, so that $c=1$, $\omega=0$. Equation (1) can then be written as

$$\frac{\partial F}{\partial t}+(k_j-\beta u_j)\frac{\partial F}{\partial u_j}+u_j\frac{\partial F}{\partial x_j}=3\beta F+q_1\frac{\partial^2 F}{\partial u_j\partial u_j} \tag{2}$$

where

$$k_j\equiv\beta\langle u_j\rangle,\qquad q_1\equiv\tfrac{1}{3}\beta_1 E$$

Hong[6] obtained the Green's function of Eq. (2) as follows:

$$G(x,u,t/x_o,u_o,t_o)=\frac{e^{3\beta t}}{8\pi^3(AC-B^2)^{3/2}}$$

$$\exp-\left\{\frac{\left[\begin{array}{l} C\left|\left(u-\dfrac{k}{\beta}\right)e^{\beta t}-\left(u_o-\dfrac{k}{\beta}\right)e^{\beta t_o}\right| \\ +2B\left[\left(u-\dfrac{k}{\beta}\right)e^{\beta t}-\left(u_o-\dfrac{k}{\beta}\right)e^{\beta t_o}\right] \\ \left[(x-x_o)+\dfrac{u-u_o}{\beta}-\dfrac{k}{\beta}(t-t_o)\right] \\ +A\left|x-x_o)+\dfrac{(u-u_o)}{\beta}-\dfrac{k}{\beta}(t-t_o)\right|^2 \end{array}\right]}{2(AC-B^2)}\right\} \tag{3}$$

where

$$A\equiv 2\int_{t_o}^{t}q_1\cdot a(\xi)\mathrm{d}\xi$$

$$B\equiv -2\int_{t_o}^{t}q_1\cdot b(\xi)\mathrm{d}\xi$$

$$C\equiv 2\int_{t_o}^{t}q_1\cdot c(\xi)\mathrm{d}\xi \tag{4}$$

$$\left(a=e^{2\beta t},\quad b=\frac{1}{\beta}e^{\beta t},\quad c=\frac{1}{\beta^2}\right)$$

Simulation of Flowfield

We assume that two turbulent streams of the same mean velocity and turbulence energy were initially separated by two infinitely long thin films (see Figs. 1-3). At $t=t_0$, these thin films are suddenly removed, and the two streams begin to mix, as shown in Fig. 3a. This instantaneous mixing phenomenon will be used to simulate the steady twin-jet mixing problem.[7-9] Assume that an observer begins to move at $t=t_0$ along the x axis with velocity $\bar{U}_m$, as shown in Fig. 3b. The observer will see the velocity profiles and other momentum quantities similar to those in Fig. 1 for steady-state twin jets. In this simulation, the x axis in the steady twin-jet problem may be considered equivalent to $\bar{U}_m t$ in the instantaneous mixing problem. This simulation will be justified by subsequently comparing the calculated results with the experimental results of Spencer's, and Lee and Harsha's data.

Source Conditions and Constructed Solution

The Green's function obtained[6] is considered to be the instantaneous point source solution of Eq. (1). In order to utilize this Green's function to construct the pdf, the source conditions have to be specified according to the given physical problem.

Assume that for $t<t_0$, the two streams are homogeneous and their pdf are Gaussian with respect to their own mean velocities and mean turbulent energies. The ambient fluid is also assumed to be homogeneous and its pdf Gaussian with respect to its own mean velocity $\bar{U}_\infty$ and turbulent energy E_{01}. Then the source conditions can be written as

$$S_{o1}=\frac{1}{(\frac{2}{3}\pi E_{o1})^{3/2}}\cdot\exp-\left[\frac{(u_o-\bar{U}_\infty)^2+v_o^2+w_o^2}{\frac{2}{3}E_{o1}}\right] \quad (5)$$

$$Y\leq-(S+D)/2,\ -(S-D)/2\leq Y\leq(S-D)/2$$

$$Y\geq(S+D)/2; \qquad x\leq o$$

$$S_{oi}=\frac{1}{(\frac{2}{3}\pi E_{oi})^{3/2}}\cdot\exp-\left[\frac{(u_o-\bar{U}_j)^2+v_o^2+w_o^2}{\frac{2}{3}E_{oi}}\right]$$

$$i=2\sim3-(S+D)/2\leq Y\leq-(S-D)/2$$

$$(S-D)/2\leq Y\leq(S+D)/2; \qquad x\leq 0 \quad (6)$$

where u_0, v_0, and w_0 are the source's instantaneous velocities in the x, y, and z directions, respectively. According to the given source conditions, pdf is constructed as follows:

$$f=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}G\cdot S_{o1}\mathrm{d}y_o\right.$$
$$-\int_{-(S+D)/2}^{-(S-D)/2}G\cdot S_{o1}\mathrm{d}y_o-\int_{(S-D)/2}^{(S+D)/2}G\cdot S_{01}\mathrm{d}y_o$$
$$\left.+\int_{-(S+D)/2}^{-(S-D)/2}G\cdot S_{02}\mathrm{d}y_o+\int_{(S-D)/2}^{(S+D)/2}G\cdot S_{03}\mathrm{d}y_o\right]$$
$$\mathrm{d}u_o\mathrm{d}v_o\mathrm{d}w_o\mathrm{d}x_o\mathrm{d}z_o \quad (7)$$

From the constructed pdf, the ensemble average $\langle Q\rangle$ can be obtained as

$$\langle Q\rangle=\int_{-\infty}^{\infty}f\cdot Q\mathrm{d}u \quad (8)$$

Then, by definition,

$$f(u)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f\mathrm{d}v\mathrm{d}w, \quad f(v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f\mathrm{d}u\mathrm{d}w \quad (9)$$

$$f(u,v)=\int_{-\infty}^{\infty}f\mathrm{d}w \quad (10)$$

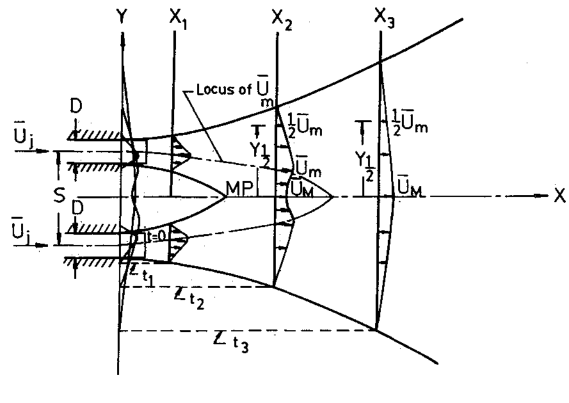

Fig. 3 Simulation of twin jets instantaneous mixing to steady twin jets mixing: a) instantaneous mixing mean velocity profiles observed at fixed x; b) mean velocity profile observed when moving with velocity U_m.

According to the analysis and simulation previously described, the x coordinate is expressed as

$$x=\int_o^t\bar{U}_m(t)\mathrm{d}t=C_2(D/x)\cdot\bar{U}_jt \quad (11)$$

According to the literature,[1,16]

$$\beta_1=\frac{E^{1/2}}{2\Lambda} \quad (12)$$

$$\beta^\nu=A'\frac{\nu}{\lambda^2} \quad (13)$$

where Λ is the characteristic length scale of the turbulence field, λ is the dissipative length scale, ν is the kinematic viscosity, and A' is a constant of order 1. By introducing the turbulent Reynolds number as

$$Re_\Lambda=\frac{E^{1/2}\Lambda}{\nu} \quad (14)$$

β^ν can be written in terms of β_1 using Eqs. (13) and (14) as

$$\beta^\nu=b'\beta_1 \quad (15)$$

where

$$b'=2\left(\frac{\Lambda}{\lambda}\right)^2\Big/Re_\Lambda \quad (16)$$

According to the previous experimental results,[17,18] the mixing layer thickness of the flowfield is approximately a linear function of X, i.e.,

$$\Lambda=C_1'X \quad (17)$$

where C_1' is a constant. From Eqs. (11), (12), and (17), β_1 can be written as

$$\beta_1=\frac{E^{1/2}}{C_1\cdot C_2(D/X)\cdot\bar{U}_jt} \quad (18)$$

where $C_1=2C_1'$ and $C_2=C_2(D/X)$. The constants C_1 and b' are characteristic properties of the turbulent field. Finally, the values of the parameters $b' \simeq 0.1$ and C_1 are chosen as listed in Table 1.

Processes of Calculation

In Hong's[6] solution of the Green's function for the kinetic equations, the ensemble averages such as $\langle u_k \rangle$ and $\langle u_k' u_k' \rangle$ appearing in the equation were assumed to be known. The equation, therefore, was linear, and the solution of f(pdf) was constructed via a linear summation of the weighted Green's functions according to the given source conditions. The ensemble averages $\langle u_k \rangle$ and $\langle u_k' u_k' \rangle$ appear in the Green's function and the constructed pdf. In order to evaluate $\langle u_k \rangle$ and $\langle u_k' u_k' \rangle$ from Eq. (8) with $Q = u_k$ and $u_k' u_k'$, one must "guess" $\langle u_k \rangle$ and $\langle u_k' u_k' \rangle$ in the right-hand-side of that equation. Therefore, an iterative scheme was needed to converge on the correct values for $\langle u_k \rangle$ and $\langle u_k' u_k' \rangle$. After $\langle u_k \rangle$ and $\langle u_k' u_k' \rangle$ are determined, all higher-order moments can be constructed from Eq. (8).

Results and Discussion

Pdf and Jpdf in Velocity Space

Solutions of Eq. (7) are shown in Figs. 4-9. In these figures, we see that the pdf's deviate from the Gaussian distribution in the region B of the twin jets. These pdf distributions show that the local turbulence property results from interaction of the two different eddy characteristics of the two jets. The interaction of these different eddy characteristics plays an important role in a multiple-jet mixing process.

The jpdf (joint probability density function) distribution in the region B at different space/width ratios $\tilde{S}$ is shown in Figs. 10-13. In region B, turbulent correlations between the two jets are strong, and the jpdf correlations further deviate from Gaussian. Therefore, the correlations between momentum fluctuations are stronger in this region. In the region downstream, the mixing is more complete, and the turbulence field is more homogeneous than that in region B (Fig. 14). The correlations in this downstream region would be weaker. This is also seen in the distribution of the Reynolds stress (Fig. 15). The previous discussion shows that one could fairly accurately predict the correlation functions from the jpdf distribution.

Pdf and Turbulent Transport

The calculated distribution of turbulent intensities is compared with the experimental data of Al-Tarafi and Miller and Comings in Figs. 16 and 17. The calculated mean velocity distributions at various x/D ratios are also compared with the experimental data of Marsters, Miller, and Comings shown in Figs. 18-20. A reasonably good agreement is shown.

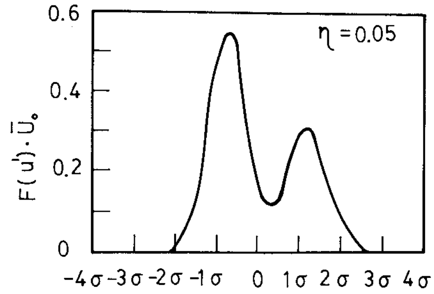

Fig. 4 Pdf, $X/D = 15$, $\eta = 0.05$, $\tilde{S} = 2$, $R = 0$.

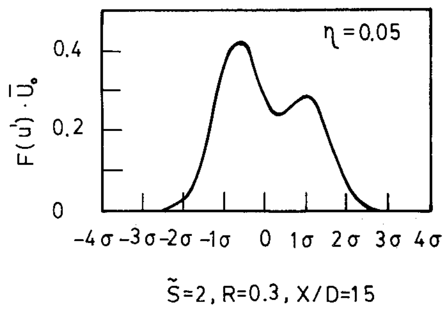

Fig. 5 Pdf, $X/D = 15$, $\eta = 0.05$, $\tilde{S} = 2$, $R = 0.3$.

Table 1 The parameters of C_1 at various $\tilde{S}$, R

$\tilde{S}=2$, $R=0$				$\tilde{S}=2$, $R=0.3$			
X/D \ Y/D	0	1	2	X/D \ Y/D	0	1	2
10	0.050	0.100	0.025	10	0.030	0.092	0.018
20	0.050	0.040	0.015	20	0.040	0.035	0.021
$\tilde{S}=6$ (without common end wall), $R=0$				$\tilde{S}=6$ (without common end wall), $R=0.3$			
X/D \ Y/D	0	3	6	X/D \ Y/D	0	3	6
20	0.037	0.090	0.027	20	0.035	0.082	0.021
30	0.041	0.042	0.020	30	0.038	0.038	0.018
$\tilde{S}=6$ (with common end wall), $R=0$				$\tilde{S}=6$ (with common end wall), $R=0.3$			
X/D \ Y/D	0	3	6	X/D \ Y/D	0	3	6
15	0.045	0.110	0.045	15	0.040	0.095	0.030
25	0.045	0.038	0.018	25	0.042	0.035	0.015
$\tilde{S}=11$, $R=0$				$\tilde{S}=11$, $R=0.3$			
X/D \ Y/D	0	6	12	X/D \ Y/D	0	6	12
25	0.032	0.085	0.022	25	0.030	0.080	0.020
50	0.060	0.070	0.028	50	0.055	0.065	0.023

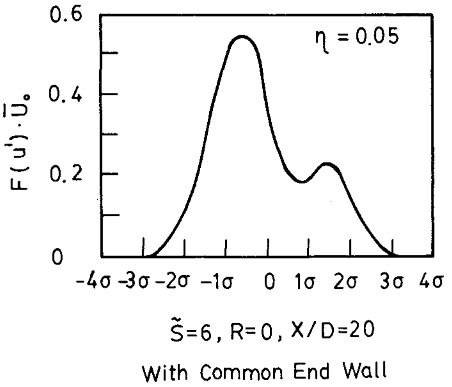

Fig. 6 Pdf, $X/D = 20$, $\eta = 0.05$, $\tilde{S} = 6$, $R = 0$, with common end wall.

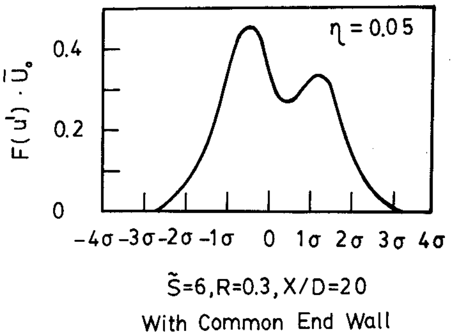

Fig. 7 Pdf, $X/D = 20$, $\eta = 0.05$, $\tilde{S} = 6$, $R = 0.3$, with common end wall.

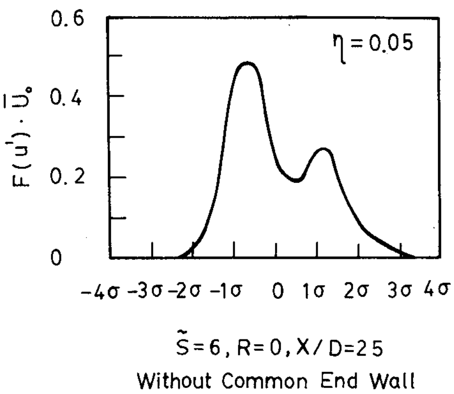

Fig. 8 Pdf, $X/D = 25$, $\eta = 0.05$, $\tilde{S} = 6$, $R = 0$, without common end wall.

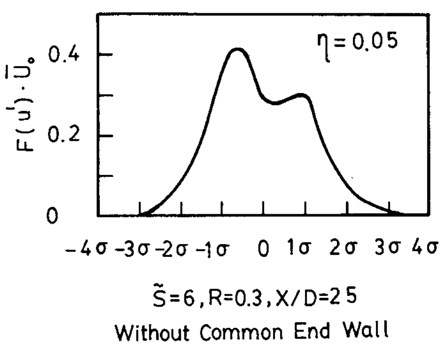

Fig. 9 Pdf, $X/D = 25$, $\eta = 0.05$, $\tilde{S} = 6$, $R = 0.3$, without common end wall.

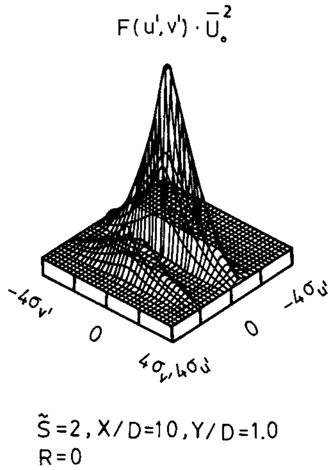

Fig. 10 Jpdf, $\tilde{S} = 2$, $X/D = 10$, $Y/D = 1.0$, $R = 0$.

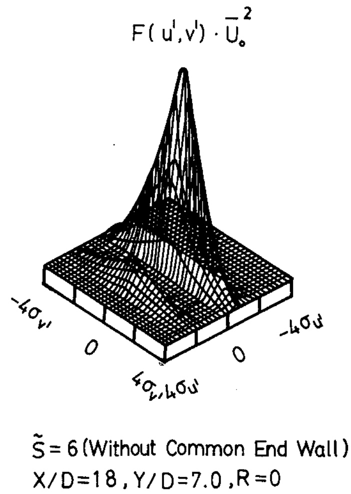

Fig. 11 Jpdf, $\tilde{S} = 6$, $X/D = 18$, $Y/D = 7.0$, $R = 0$, without common end wall.

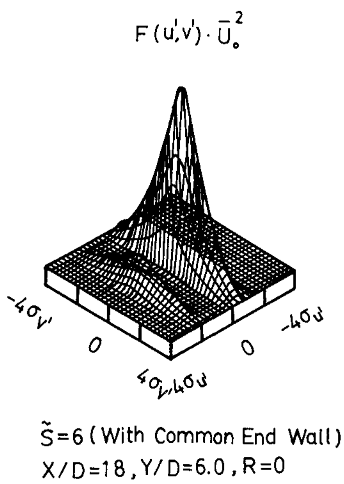

Fig. 12 Jpdf, $\tilde{S} = 6$, $X/D = 18$, $Y/D = 6.0$, $R = 0$, with common end wall.

The $f(u')$ distribution shown in Fig. 21 can well explain the turbulent intensity $\sqrt{\overline{u'^2}}$ distributions shown in Fig. 17 at each corresponding location in the mixing layer. It is noted that the $f(u')$ distribution is wider (Fig. 21 for $Y/S = 0.1$) around the interaction region between two jets, and this implies high turbulent intensity, as shown in Fig. 17. The wider the span of $f(u')$, the more the fluid elements fluctuate away

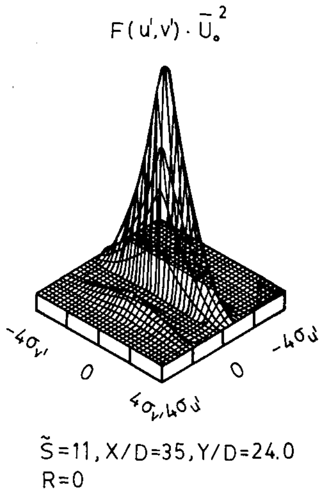

Fig. 13 Jpdf, $\tilde{S}=11$, $X/D=35$, $Y/D=24.0$, $R=0$.

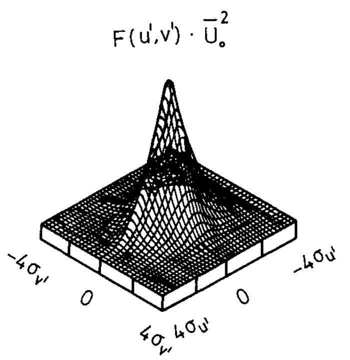

Fig. 14 Jpdf.

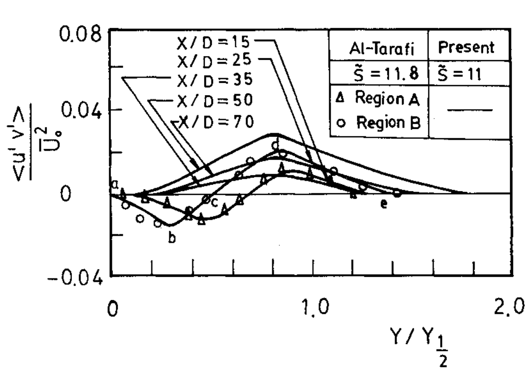

Fig. 15 Reynolds shear stress distributions.

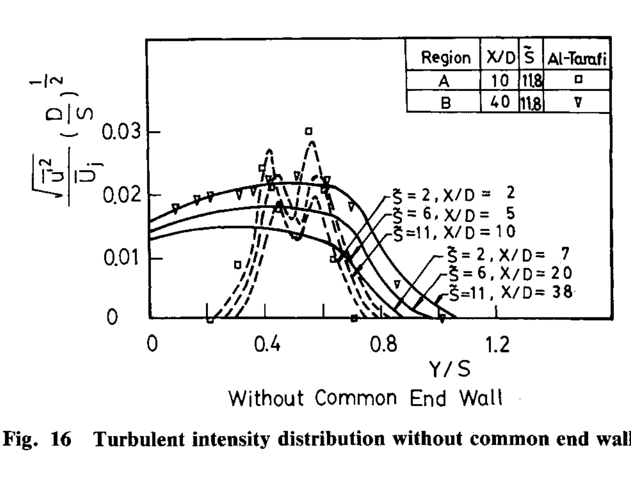

Fig. 16 Turbulent intensity distribution without common end wall.

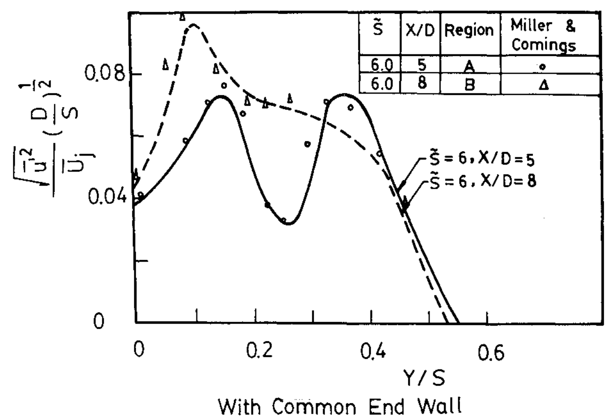

Fig. 17 Turbulent intensity distribution with common end wall.

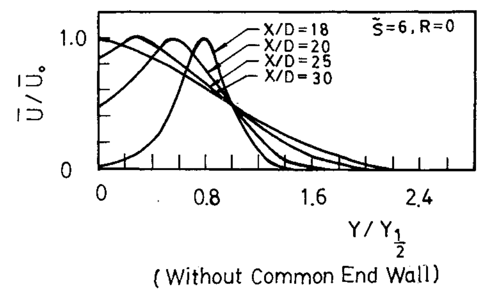

Fig. 18 The axial component average velocity, $\tilde{S}=6$, $R=0$, without common end wall.

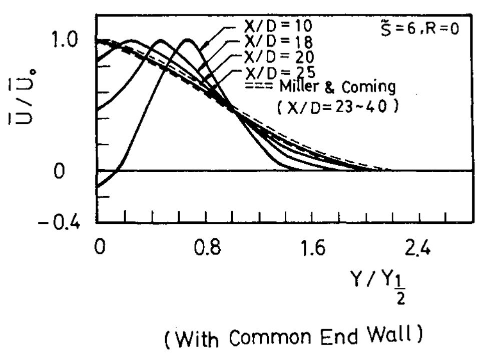

Fig. 19 The axial component average velocity, $\tilde{S}=6$, $R=0$.

from the mean, bringing about higher turbulent energy and intensity. It can also be concluded that the mixing is more intense in the interaction region between two jets, which is physically expected.

The calculated Reynolds stress distributions at various X/D ratios are compared with the experimental data of Al-Tarafi shown in Fig. 15. Distributions of the joint probability density function of $f(u',v')$ corresponding to the case of $X/D=25$ in Fig. 15 are shown in Fig. 22. By comparing Fig. 22 with Fig. 15, one can easily see that at $Y/Y_{½}$ locations of a, c, and e, the jpdf's of u' and v' are closely Gaussian and the $\langle u'v' \rangle$ correlations are weaker. At locations b and d, the correlations are stronger and the values of $\langle u'v' \rangle$ are higher due to the distribution of jpdf, also, $f(u',v')$ is less Gaussian. At location d in Fig. 22, the $f(u',v')$ is skewed to both negative u' and v' sides, and hence, the correlation of $\langle u'v' \rangle$ turns out to be positive. In Fig. 22b, the $f(u',v')$ distribution is skewed to the positive v' and negative u' sides; therefore, the corresponding $\langle u'v' \rangle$ correlation in Fig. 15 turns out to be negative. Now the description of the relationship between jpdf, $f(u',v')$, and the cross-correlation of $\langle u'v' \rangle$ is complete.

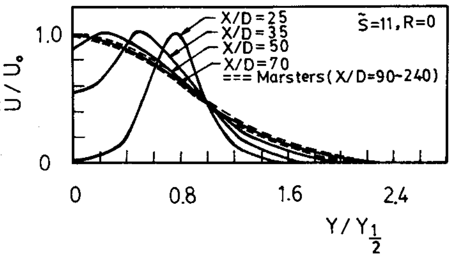

Fig. 20 The axial component average velocity, $\tilde{S}=11$, $R=0$.

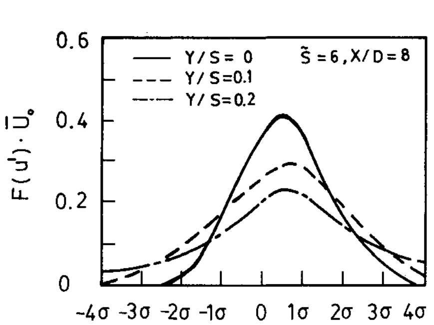

Fig. 21 Pdf.

One of the benefits of the present approach is that it can generate the various order correlations without using the conventional eddy viscosities. The third-order correlations of turbulent energy transport terms $\langle u'^2 v' \rangle$ are calculated as shown in Fig. 23. The distribution of $f(v')$ (Fig. 24) displays Gaussian behavior in the central region ($Y/Y_{1/2}=0$) and is skewed to positive or negative velocity fluctuation when $Y/Y_{1/2}>0$ or $Y/Y_{1/2}<0$, respectively. The distribution of $f(v')$ also explains the transport of turbulent energy in the v' direction (Y direction). A Gaussian distribution of velocity fluctuations of v' has zero transport of $\langle u'^2 \rangle$ in the Y direction. The skewed $f(v')$ distributions of Fig. 24 at the two other locations possess nonzero odd-order moments and hence imply nonzero turbulent energy transport of $\langle u'^2 \rangle$.

The calculation of third-order moments via conventional turbulent modeling would be very difficult and tedious. First, one would have to construct the moment equations for the third-order correlations and then try to close the fourth-order moments and many other fluctuation gradient correlations appearing in the equation. Numerous "constants" or transport coefficients would emerge, and many heavy experiments would be needed to justify those undetermined constants. The present kinetic theory approach along with the modified Green's function method seems better justified in the sense of generating the moments, and the mixing mechanism seems better understood via the revealed pdf and jpdf.

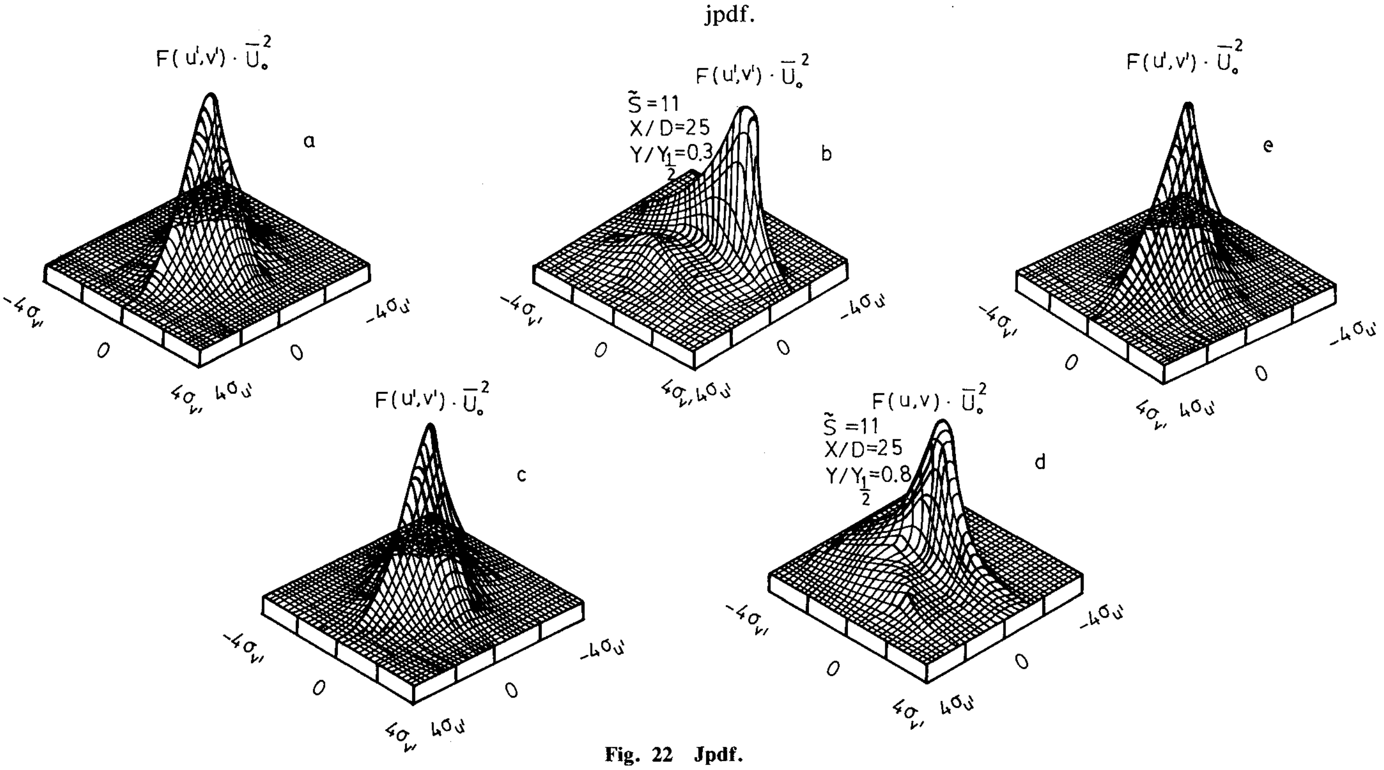

Fig. 22 Jpdf.

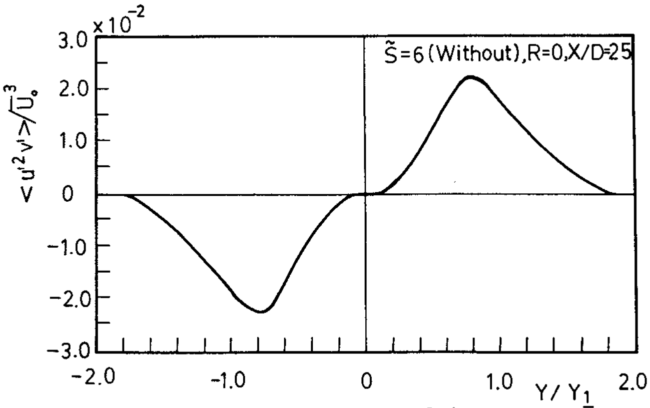

Fig. 23 Third-order correlations.

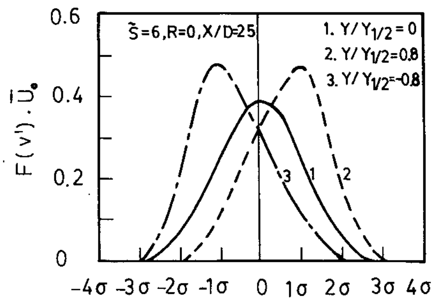

Fig. 24 Pdf.

Concluding Remarks

1) The mean flow quantities for the incompressible mixing of twin plane jets have been examined and calculated by the present kinetic theory approach. The present approach seems warranted when compared to the calculated results and experimental data.

2) The present pdf approach and the Green's function employed seem more general in the sense of generating the moments (any higher-order correlations).

3) The mixing mechanism and the Reynolds stress were fairly better understood via the corresponding revealed jpdf (joint probability density function).

4) Some inside structure of the transport phenomena of the second-order correlations (turbulent energies) were better described in velocity space via the revealed pdf in the transported direction.

5) Any higher-order correlations or higher-order turbulent transport phenomena can be predicted and explained in a similar way as mentioned in conclusions 3 and 4.

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References

[1]Chung, P. M., "Turbulent Chemically Reacting Flows," Rept. TR-1001(S2855-20)-5, The Aerospace Corp., San Bernardino, CA, 1967.

[2]Chung, P. M., "On the Development of Diffusion Flame in Homologous Turbulent Shear Flows," AIAA Paper 70-722, 1970.

[3]Chung, P. M., "Turbulence Description of Couette Flow," *The Physics of Fluids*, Vol 16, 1973, p. 980.

[4]Bywater, R. J. and Chung, P. M., "Turbulent Flow Fields with Two Dynamically Significant Scales," AIAA Paper 73-646, 1973.

[5]Lin, C. Y. and Lees, L., *Kinetic Theory Description of Plane Compressible Couette Flow, Rarefied Gasdynamics*, edited by L. A. P. Talbot, 1961.

[6]Hong, Z. C., "Turbulent Chemically Reacting Flows According to a Kinetic Theory," Ph.D. Thesis, Dept. of Energy Engineering, Univ. of Illinois, Chicago Circle, IL, 1975.

[7]Hong, Z. C. and Lai, Z. C., "On the Mixing Analysis of a Free Turbulence Shear Layer," *Proceedings of the 1st Conference on Theoretical and Applied Mechanics*, Republic of China, 1977.

[8]Wang, J. H., "Plane Jet Turbulent Mixing and Combustion," Master Thesis, Dept. of Mechanical Engineering, National Taiwan Univ., Republic of China, 1978.

[9]Hong, Z. C. and Chen, L., "HF Chemical Laser Properties Analysis in a Turbulent Plane Jet," *Journal of Chinese Society of Mechanical Engineers*, Vol. 5, No. 1, 1984, pp. 45–66.

[10]Miller, D. R. and Comings, E. W., "Force-Momentum Fields in a Dual-Jet Flow," *Journal of Fluid Mechanics*, Vol. 7, 1960, pp. 237–256.

[11]Tanaka, E., "The Interference of Two-Dimensional Parallel Jets," *Bulletin of the Japan Society of Mechanical Engineers*, Vol. 17, 1974, pp. 920–927.

[12]Marsters, G. F., "Interaction of Two Plane, Parallel Jets," *AIAA Journal*, Vol. 15, Dec. 1977, pp. 1756–1762.

[13]Al-Tarafi, A. H. A., "Flow Structure of Plane Turbulent Twin Jets," Ph.D. Thesis, Queen Mary College, Univ. of London, 1976.

[14]Bywater, R. J., "Velocity Space Description of Certain Turbulent Free Shear Flow Characteristics," *AIAA Journal*, Vol. 19, Aug. 1981, pp. 969–975.

[15]Bywater, R. J., "Numerical Solutions of a Reduced pdf Model for Turbulent Diffusion Flames," *AIAA Journal*, Vol. 20, June 1982, pp. 824–830.

[16]Batchelor, G. K., "The Theory of Homogeneous Turbulence," Cambridge University Press, Cambridge, 1960.

[17]Lee, S. C. and Harsha, P. T., "Use of Turbulent Kinetic Energy in Free Mixing Studies," *AIAA Journal*, 1970, pp. 1026–1038.

[18]Spencer, B. W., "Statistical Investigation of Turbulent Velocity and Pressure Fields in a Two-Stream Mixing Layer," Ph.D. Thesis, Univ. of Illinois, Urbana, IL, 1970.